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Exchange rate volatility and deviations from unbiasedness in a cash-in-advance model

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This paper examines time-series properties of exchange rate changes, the forward premium and the forward bias in the context of a variant of Svensson's cash-in-advance model. The model is solved and simulated using realistic forcing processes whose law of motion is estimated from U.S.–Japan data and then approximated by a Markov chain. Although method of moments estimation shows that the over-identifying restrictions implied by the model are not rejected, it fails dramatically in producing a sufficiently variable risk premium on forward market speculation. This result is robust to various perturbations to the model's parameters, forcing processes and preference structure. The model also fails to match exchange rate and forward premium volatility simultaneously.

1. Introduction

The floating exchange rate period has generated numerous puzzling regularities. One of the stylized facts is the high variability of exchange rate movements which has raised concern about 'excessive' volatility of foreign exchange rates. Another well-known puzzle is the clear rejection of the 'unbiasedness hypothesis', which postulates that the forward rate is an unbiased predictor of the future spot rate. Attempts to model the deviation from unbiasedness as a time-varying risk premium in the context of simple general equilibrium models have not met with much empirical success. The variability of the risk premium predicted by the models is typically orders of magnitude smaller than what is observed in the data.

Most of these studies are based upon the two-country, cash-in-advance

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(CIA) model of Lucas (1982).[1] The goal of this paper is to judge the 'excessiveness' of both exchange rate volatility and deviations from unbiasedness from the perspective of Svensson's (1985a) two-country CIA model. The Svensson model differs from the Lucas model primarily in the timing of information arrival. In the Lucas model, all uncertainty is resolved at the moment when consumers choose the money holdings with which they will buy consumption goods and, given positive nominal interest rates, the CIA constraints are always binding. In the Svensson model non-binding CIA constraints with positive interest rates are possible as consumers decide on their cash holdings before the state is revealed. As a consequence, velocity directly enters the exchange rate and risk premium expressions and is potentially an important factor in their determination which the Lucas model lacks. The particular timing of the Svensson model also induces a wedge between the marginal utility of wealth and the marginal utility of consumption except when the CIA constraints are slack. As a consequence, real interest rates depend on monetary policy, which is not true in the Lucas model. Moreover, as agents have to trade currencies before the state is revealed, the Svensson model also induces a forward-looking spot exchange rate. This implies that, even with binding CIA constraints, the exchange rate and risk premium differ across the models.

This paper differs from related studies in other aspects as well. First, rather than imposing the real world exchange rate process as Macklem (1991) and Backus et al. (1992) do, I solve for exchange rate changes and its moments as a function of the exogenous processes (money and endowment shocks) and model parameters. A focus on several exchange rate moments, jointly with the issue of risk premium variability, reveals several trade-offs in the model's ability to match different aspects of the data.

Second, the law of motion for the forcing processes is estimated from a vector autoregression (VAR) on U.S.–Japanese data on money and consumption. This law of motion is approximated by a first-order Markov chain using a discretization technique developed by Tauchen and Hussey (1991). In the data, there is correlation between monetary and real shocks, and Engel (1992), in the context of the Lucas model, stresses the importance of these comovements in the determination of the risk premium. The Markov chain replicates the actual correlation structure of the data.

Third, rather than simulating the model at pre-specified preference parameters, I estimate the preference parameters with a variant of Hansen's (1982) General Method of Moments (GMM). The estimation technique minimizes a weighted sum of the deviations between the sample moments and numerically obtained model moments of exchange rate changes, the

[1]See the studies of Canova and Marrinan (1991), Macklem (1991), Backus et al. (1992) and Engel (1992). Hakkio and Sibert (1991) perform simulations of an overlapping-generations model and also find the variability of the model's risk premium to be very small.

forward premium and the forward bias.[2] Mark (1985) and Hodrick (1989) perform Euler equation tests of a representative agent economy, using forward market returns. The point estimates of the coefficient of relative risk aversion are large and the standard errors encompass a wide range of possible values. The curvature parameters obtained here are more reasonable and more precisely estimated. This suggests that the exchange rate moments employed here may be more informative about preferences. Although I fail to reject the over-identifying restrictions, the implied moments reveal some dimensions along which the model fails. In particular, the implied risk premium is far from variable enough.

The paper is organized as follows. Section 2 contains a brief discussion of a growth version of the Svensson model and a description of a solution technique that allows the use of realistic forcing processes and does not impose binding CIA constraints. Solutions for endogenous variables for several preference specifications are presented.

In section 3 the estimation methodology and results are discussed. The empirical results are shown to be robust to a number of perturbations to the forcing processes and parameters. One robust finding is the failure of the international Svensson model to deliver variable velocity. This generalizes the simulation results of Hodrick et al. (1991), who examine the closed economy Svensson (1985b) model. They find the CIA constraint to be always binding for a wide range of parameter values when the forcing processes are calibrated to correspond with U.S. data on consumption and money growth.

The results are further examined in section 4. The failure to match risk premium variability is interpreted employing Hansen–Jagannathan (1991) bounds. The latter are bounds on intertemporal marginal rates of substitution (IMRS) that can be derived from asset market data. Backus et al. (1992) note that habit-forming utility increases the variability of the risk premium. I show that this is also true in the Svensson economy but that there is a trade-off between matching exchange rate and risk premium volatility. Habit persistence also leads to many states with a large precautionary demand for money and hence non-binding CIA constraints. Finally, I address one of the obvious limitations that the Svensson model shares with most models in this literature: the implict assumption of purchasing power parity (PPP). As PPP is grossly violated in the data, it is important to establish whether the lack of a channel to generate PPP deviations might partially explain the poor performance of this class of models with respect to the forward market risk

[2]Bansal (1989) estimates the parameters of a transaction cost model with standard GMM, but he focuses on the terms of trade and investigates the forward market risk premium implied by the model solely in terms of Hansen–Jagannathan bounds (see below). Canova and Marrinan (1991) used a Simulated Moments Estimator to estimate the parameters of a Lucas-type model using moments of the forward bias.

premium. The concluding section sums up and discusses some further research possibilities.

2. Svensson's two-country model

2.1. First-order conditions and solution algorithms

In Svensson's model an infinitely lived representative consumer in each of two countries maximizes a time-separable utility function. The representative agent enters each period with predetermined holdings of home and foreign money and of the assets that are traded in the economy. He then learns the current state and purchases home and foreign goods with home and foreign currency, respectively. At the end of the period, there is an asset market in which currencies and assets are traded and at which time monetary transfers are received. Markets are perfectly competitive and agents have rational expectations.

Let x_t^s (y_t^s) denote the home (foreign) country's stochastic, non-storable endowment of goods and M_t^s and N_t^s the home and foreign money supply at time t. Since growth rates will follow a Markov chain, I use mc_t to denote the vector that contains the (gross) rates of the variables described above, i.e. $mc_t=[gx_t, gy_t, \omega_t, \omega_t^*]'$, where $gi_t=i_t/i_{t-1}$ $(i=x,y)$ and $\omega_t=M_{t+1}^s/M_t^s$, $\omega_t^*=N_{t+1}^s/N_t^s$. The state vector for this economy is then given by $\theta_t=[x_t^s, y_t^s, M_t^s, N_t^s, mc_t']'$.

Agents can purchase/sell claims to all of the endowment processes and to the monetary tranfers. There is one perfectly divisible share of each asset. Asset prices are given by $Q_t=[Q_t^x, Q_t^y, Q_t^M, Q_t^N]'$ and asset holdings are summarized by $\alpha_t=[\alpha_t^x, \alpha_t^y, \alpha_t^M, \alpha_t^N]'$ for the home consumer (α_t^* for the foreign consumer). The dividends are nominal and expressed in the currency of the home country, i.e. $D_t=[P_t^x x_t^s, S_t P_t^y y_t^s, (\omega_t-1)M_t^s, S_t(\omega_t^*-1)N_t^s]'$ with S_t indicating the level of the spot exchange rate between the two currencies (i.e. home currency per unit of the foreign currency) and (P_t^x, P_t^y) denoting the respective goods prices in home and foreign currency units.

The home consumer's decision problem can now be described by the following equations, representing respectively his preferences, the CIA constraints and the budget constraint:

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U(x_t^{\mathrm{d}}, y_t^{\mathrm{d}}), \tag{1}$$

$$P_t^x x_t^{\mathrm{d}} \leqq M_t^{\mathrm{d}}, \qquad S_t P_t^y y_t^{\mathrm{d}} \leqq S_t N_t^{\mathrm{d}}, \tag{2}$$

$$\begin{aligned} M_{t+1}^{\mathrm{d}} + S_t N_{t+1}^{\mathrm{d}} + \alpha_{t+1}' Q_t &\leqq \alpha_t'(Q_t + D_t) + (M_t^{\mathrm{d}} - P_t^x x_t^{\mathrm{d}}) \\ &\quad + S_t(N_t^{\mathrm{d}} - P_t^y y_t^{\mathrm{d}}). \end{aligned} \tag{3}$$

By adding the value of the cash goods consumption on both sides, the right-hand side of the modified eq. (3) defines nominal wealth. It consists of the proceeds of the sale of the asset holdings, of dividends and of money holdings. Note that all constraints are expressed in units of the home currency. I only investigate the standard perfectly pooled equilibrium as defined in Lucas (1982).[3]

Because of the law of one price, the exchange rate equates the value of a foreign currency unit in today's asset market with the value of a home currency unit in today's asset market. Since a currency unit acquired in today's asset market can only be used for consumption next period, the value of a home (foreign) currency unit today equals the expected marginal utility of the home (foreign) good per unit of the home (foreign) currency. Hence, the exchange rate is a forward-looking asset price in the Svensson model:

$$S_t = \frac{\mathrm{E}_t\left[\dfrac{U_2(x^s_{t+1}, y^s_{t+1})}{P^y_{t+1}}\right]}{\mathrm{E}_t\left[\dfrac{U_1(x^s_{t+1}, y^s_{t+1})}{P^x_{t+1}}\right]}, \tag{4}$$

where the subscripts on U denote partial derivatives.

Note that the expected marginal utility of the home good per unit of the home currency also equals the Lagrange multiplier on the budget constraint λ_t as this represents the marginal utility of wealth in home currency terms. The nominal (home currency) intertemporal marginal rate of substitution (IMRS), denoted by n_{t+1}, is the ratio of the discounted value of a unit of the home currency tomorrow $(\beta\lambda_{t+1})$ and the value of a home currency unit today (λ_t). Therefore, it is given by

$$n_{t+1} = \beta\frac{\lambda_{t+1}}{\lambda_t} = \beta\frac{\mathrm{E}_{t+1}\left[\dfrac{U_1(x^s_{t+2}, y^s_{t+2})}{P^x_{t+2}}\right]}{\mathrm{E}_t\left[\dfrac{U_1(x^s_{t+1}, y^s_{t+1})}{P^x_{t+1}}\right]}, \tag{5}$$

To solve the model without assuming the CIA constraints to be binding, I adapt a technique from Giovannini and Labadie (1991). The crucial step in the solution algorithm is to solve for the inverse of velocity for home and foreign money, denoted respectively by $K(\theta_t)$ and $K^*(\theta_t)$. Manipulation of the first-order conditions yields

[3]A formal statement of the optimization problem and the first-order conditions is given in an unpublished appendix, which is available upon request.

$$K(\theta_t)=\max\left\{1,\beta \mathrm{E}_t\left[K(\theta_{t+1})\,\frac{gx_{t+1}}{\omega_t}\,\frac{U_1(x^s_{t+1},y^s_{t+1})}{U_1(x^s_t,y^s_t)}\right]\right\}. \tag{6}$$

An analogous expression can be derived for $K^*(\theta_t)$. It can be shown that, under suitable conditions, the mapping defined in (6) is a contraction.[4] By definition, $P^x_t x^s_t K(\theta_t)=M^s_t$ so that $K(\theta_t)$ [and analogously $K^*(\theta_t)$] and the state vector determine prices, which in turn can be combined with the state vector to yield solutions for the exchange rate and the IMRS according to eqs. (4) and (5).

The forward rate, F_t, and the forward premium, $FP_t=(F_t-S_t)/S_t$, can be deduced from covered interest rate parity. Nominal interest rates are found by pricing a nominal bond. The price of such a bond, yielding one unit of the home (foreign) currency next period equals the conditional expected value of the home (foreign) nominal IMRS. The foreign IMRS is $(S_{t+1}/S_t)\,n_{t+1}$. Letting i_t and i^*_t denote home and foreign interest rates, it follows:

$$\begin{aligned}
&\frac{F_t}{S_t}(1+i^*_t)=1+i_t,\\
&1+i_t=\{\mathrm{E}_t[n_{t+1}]\}^{-1}, \qquad\qquad (7)\\
&1+i^*_t=\left\{\mathrm{E}_t\left[n_{t+1}\frac{S_{t+1}}{S_t}\right]\right\}^{-1}.
\end{aligned}$$

The variables of interest are now completely characterized. Currency depreciation, $DS_{t+1}=(S_{t+1}-S_t)/S_t$, can be found from eq. (4); the normalized forward bias, $FB_{t+1}=(S_{t+1}-F_t)/S_t=DS_{t+1}-FP_t$, from eqs. (4) and (7). The predictable component in the forward bias, $\mathrm{E}_t[FB_{t+1}]=\mathrm{E}_t[DS_{t+1}]-FP_t$, equals zero when the unbiasedness hypothesis holds and is usually termed the 'risk premium in the forward market'. I will denote it by RP_t.[5]

2.2. Endogenous variables for homothetic and addilog utility

Two widely used preference specifications are

[4]The proof is similar to the proof in Giovannini and Labadie (1991) and is omitted.

[5]The nominal risk premium as defined here might not be equal zero, even if consumers are risk neutral, because of the stochastic inflation effect. Macklem (1991) finds that the stochastic inflation effect in the Lucas model is only relatively important at low levels of risk aversion and in general remains small. Engel (1992) discusses the problems associated with appropriately defining the risk premium in the forward market in a multigood economy.

$$U(x_t, y_t) = \frac{(x_t^\delta y_t^{1-\delta})^{1-\alpha}}{1-\alpha},$$

$$U(x_t, y_t) = \frac{x_t^{1-\alpha}}{1-\alpha} + \frac{y_t^{1-\gamma}}{1-\gamma}. \tag{8}$$

The first utility function is homothetic and strictly concave in its arguments for δ in $(0, 1)$ and α strictly positive. The intratemporal elasticity of substitution between x and y is 1. The intertemporal elasticity of substitution with respect to the composite good, $x_t^\delta y_t^{(1-\delta)}$, is $(1/\alpha)$. Addilog preferences are separable in the two goods. When α and γ go to 1 they reduce to logarithmic preferences. Strict positivity of α and γ ensures strict concavity, and the intertemporal elasticity of substitution is $1/\alpha$ for the home good and $1/\gamma$ for the foreign good.[6]

Expressions for the exchange rate and the nominal IMRS under these preferences are summarized in table 1. For comparison, I include the expressions for the Lucas model. With homothetic preferences and binding CIA constraints, the exchange rate in the Svensson model reduces to the exchange rate in the Lucas model. Moreover, the models reproduce the very simplest version of a monetary exchange rate model.

Both for addilog and homothetic utility, the inverse of velocity of home or foreign money constitutes an additional source of variation which the Lucas model lacks. Moreover, the particular timing of the Svensson model implies that expected marginal utilities determine the exchange rate and the nominal IMRS. Since the risk premium depends both on expected exchange rate changes and the home and foreign IMRS, it differs from the risk premium in the Lucas model even when the CIA constraints bind.

Note that DS_{t+1} and n_{t+1} have been written in terms of the sub-state vector mc_t. As this sub-state vector contains stationary growth rates, DS_{t+1}, n_{t+1} and the other endogenous variables derived from them will be stationary too. The law of motion for mc_t then completely defines the stochastic structure of the model and allows the computation of the (joint) population moments of the stationary endogenous series.

2.3. *The law of motion for the forcing processes*

To implement the solution procedure, I determine the law of motion of the state variable mc_t and then convert it into a discrete Markov chain. To measure home and foreign transactions monies on a quarterly basis, I average end-of-month U.S. and Japanese stocks obtained from International

[6]Both homothetic utility and addilog utility when α is restricted to equal γ are special cases of the general multi-good utility function defined in Eichenbaum and Hansen (1990).

Table 1

Endogenous variables in the Lucas and Svensson models.

Lucas	Homothetic	Addilog
$DS^{\mathrm{L}}_{t+1}+1=$	$\dfrac{\omega_{t+1}}{\omega^*_{t+1}}$	$\dfrac{\omega_{t+1}}{\omega^*_{t+1}}\dfrac{gy^{1-\gamma}_{t+1}}{gx^{1-\alpha}_{t+1}}$
$n^{\mathrm{L}}_{t+1}=$	$\beta\dfrac{(gx^{\delta_F}_{t+1}gy^{\delta_F}_{t+1})^{1-\alpha}}{\omega_{t+1}}$	$\beta\dfrac{gx^{1-\alpha}_{t+1}}{\omega_{t+1}}$

Svensson–homothetic

$$DS^{\mathrm{S}}_{t+1}+1=\frac{\omega_{t+1}}{\omega^*_{t+1}}\,\frac{\mathrm{E}_{t+1}[(gx^{\delta_x}_{t+2}gy^{\delta_y}_{t+2})^{1-\alpha}K^*_{t+2}]}{\mathrm{E}_t[(gx^{\delta_x}_{t+1}gy^{\delta_y}_{t+1})^{1-\alpha}K^*_{t+1}]}\,\frac{\mathrm{E}_t[(gx^{\delta_x}_{t+1}gy^{\delta_y}_{t+1})^{1-\alpha}K_{t+1}]}{\mathrm{E}_{t+1}[(gx^{\delta_x}_{t+2}gy^{\delta_y}_{t+2})^{1-\alpha}K_{t+2}]},$$

$$DS^{\mathrm{SB}}_{t+1}+1=\frac{\omega_{t+1}}{\omega^*_{t+1}},$$

$$n^{\mathrm{s}}_{t+1}=\beta\frac{\mathrm{E}_{t+1}[(gy^{\delta_x}_{t+2}gy^{\delta_y}_{t+2})^{1-\alpha}K_{t+2}]}{\mathrm{E}_t[(gx^{\delta_x}_{t+1}gy^{\delta_y}_{t+1})^{1-\alpha}K_{t+1}]}\,\frac{(gx^{\delta_x}_{t+1}gy^{\delta_y}_{t+1})^{1-\alpha}}{\omega_{t+1}}.$$

Svensson–addilog

$$DS^{\mathrm{S}}_{t+1}+1=\frac{\omega_{t+1}}{\omega^*_{t+1}}\,\frac{\mathrm{E}_{t+1}[gy^{1-\gamma}_{t+2}K^*_{t+2}]}{\mathrm{E}_t[gy^{1-\gamma}_{t+1}K^*_{t+1}]}\,\frac{\mathrm{E}_t[gx^{1-\alpha}_{t+1}K_{t+1}]}{\mathrm{E}_{t+1}[gx^{1-\alpha}_{t+2}K_{t+2}]}\,\frac{gy^{1-\gamma}_{t+1}}{gx^{1-\alpha}_{t+1}},$$

$$n^{\mathrm{S}}_{t+1}=\beta\frac{\mathrm{E}_{t+1}[gx^{1-\alpha}_{t+2}K_{t+2}]}{\mathrm{E}_t[gx^{1-\alpha}_{t+1}K_{t+1}]}\,\frac{gx^{1-\alpha}_{t+1}}{\omega_{t+1}}.$$

Notes: The L-superscript denotes the Lucas model, the S-superscript the Svensson model and SB the Svensson model with binding CIA constraints. $K_t(K^*_t)$ represent $K(\theta_t)(K(\theta^*_t))$. Their values in terms of the exogenous processes are completely determined by eq. (6) in the text.

Financial Statistics (IFS) of the IMF (the sum of the money and quasi-money series). As an empirical proxy to the endowment series, I use data on consumption of non-durables and services from the OECD Quarterly National Accounts. All the series are expressed per capita and are deseasonalized. More details can be found in a data appendix.[7]

The joint distribution of the exogenous variables is assumed to be appropriately described by a finite-order vector autoregression (VAR) with Gaussian errors. Table 2 reports the VAR estimation results. To conform with the Svensson timing, the money growth series were lagged one period,

[7]The use of quarterly data stems from the fact that decent empirical measures for the forcing processes of the theoretical models are only available at the quarterly level. In Bekaert (1992), I explore the effects of temporal aggregation in a dynamic economy similar to the one analyzed here.

Table 2

Estimated VAR and its Markov counterpart for the Svensson model. Sample period: 1975:2–1989:4.

Panel A: Test of VAR length

	Akaike criterion	Schwarz criterion
Order 1	−40.52	−39.94
Order 2	−40.25	−39.09
Order 3	−40.48	−38.74

Likelihood ratio tests

1 vs. 2	14.23 (0.581)
2 vs. 3	34.33 (0.005)

Panel B: Estimates and induced coefficients

	Constant	USM	USC	JPM	JPC	Q2(4)	BJ
USM	0.460	0.698	−0.178	0.006	0.019	3.112	4.423
$R^2=0.427$	(0.247)	(0.095)	(0.166)	(0.115)	(0.098)	(0.539)	(0.110)
	0.487	0.653	−0.156	0.005	0.017		
USC	0.482	0.210	0.191	0.052	0.063	2.447	1.820
$R^2=0.283$	(0.156)	(0.053)	(0.100)	(0.056)	(0.048)	(0.654)	(0.402)
	0.514	0.201	0.177	0.048	0.058		
JPM	0.210	−0.127	0.624	0.260	0.049	1.842	1.255
$R^2=0.131$	(0.252)	(0.096)	(0.237)	(0.112)	(0.104)	(0.765)	(0.345)
	0.215	−0.119	0.615	0.257	0.049		
JPC	1.052	0.046	−0.028	0.144	−0.207	2.295	0.345
$R^2=-0.013$	(0.343)	(0.148)	(0.280)	(0.154)	(0.137)	(0.682)	(0.842)
	1.050	0.048	−0.028	0.144	−0.206		

Panel C: Unconditional means and correlation matrix of the residuals

	USM	USC	JPM	JPC	Means
USM	0.00630	−0.018	−0.081	−0.016	1.016
	0.00590	−0.017	−0.077	−0.015	1.016
USC	-	0.00327	−0.088	−0.011	1.004
		0.00313	−0.086	−0.011	1.004
JPM	–	-	0.00700	0.011	1.022
			0.00693	0.011	1.022
JPC	–	–	–	0.00850	1.009
				0.00849	1.009

Notes: In Panel A, the likelihood ratio statistics incorporate the degrees of freedom correction recommended by Sims (1980). Marginal levels of significance are given in parentheses. In Panel B, M stands for the (M − 2) money measure (per capita) in gross growth rates, C for growth rates of real consumption per capita, US for United States and JP for Japan. In Panel B, the first line for each variable reports the OLS parameter estimates with heteroskedasticity consistent standard errors on the second line. The third line contains the induced parameters computed from the approximating Markov chain (see text). R^2 is the adjusted R^2 (coefficient of determination). The last column reports the standard Ljung–Box statistic including four autocorrelations of the *squared* residuals. Under the null of conditional homoskedasticity the statistics should have a $\chi^2(4)$-distribution. The last column reports the Bera–Jarque (1982) test for normality which has a $\chi^2(2)$-distribution under the null. P-values are reported between parentheses. In Panel C, the first line refers to the original VAR estimation, the second line to the induced VAR from the Markov chain. The diagonal elements are standard deviations of the corresponding residuals. The last column reports the unconditional means implied by the VAR and the approximating Markov chain.

so that $\omega_t = M^s_{t+1}/M^s_t$ and $\omega^*_t = N^s_{t+1}/N^s_t$ enter jointly with $gx_t = x_t/x_{t-1}$ and $gy_t = y_t/y_{t-1}$. I assess the order of the VAR with likelihood ratio tests and Schwartz and Akaike criteria, which are reported in Panel A of table 2. The Akaike and Schwarz criteria both select the first-order VAR and a likelihood ratio test also does not reject the restrictions of the first-order VAR vs. a second-order VAR. I therefore choose to work with a first-order VAR. The parameter estimates are given in Panel B. While there are few significant cross-country linkages, the predictability of the U.S. series and Japanese money is strong. Japanese consumption, on the other hand, is not predictable by any of the VAR variables. Tests for conditional homoskedasticity and normality support the assumption of homoskedastic normal errors.

The next step is to approximate this continuous state space economy with an auxiliary, discrete economy. Tauchen and Hussey (1991) describe a procedure for approximating integral operators which can be used to convert a continuous distribution into a discrete Markov chain. The multivariate normal distribution, implicit in the above VAR estimation, can be rewritten as the product of univariate normal densities with an appropriate change of variables. The univariate densities are then approximated with a Gaussian quadrature rule. The state space therefore expands exponentially. I choose 3 states of nature for each variable which results in a total of $3^4 = 81$ states. Although the approximation gets better with finer state spaces, it is already quite accurate for very coarse state spaces. One way to judge the accuracy of the approximation is to compare the original VAR estimates with the autoregressive parameters and the residual covariances induced by the Markov chain. As Panel B of table 2 indicates, all induced parameter estimates are within at most half a standard error of the original estimates. Panel C likewise shows that the Markov chain replicates the correlation structure of the actual shocks in the economy.

The discretization procedure yields a vector of 81 state values for the forcing variables, a vector of stationary probabilities and a transition probability matrix, which are sufficient to solve the models, to evaluate the conditional expectations in the expressions for the forward premium and risk premium, and to compute moments for the endogenous variables of interest.

3. Estimation and empirical results

3.1. Econometric methodology

Denote the utility parameters by ϕ_1 and the parameters governing the law of motion of the forcing processes, i.e. the VAR parameters, by ϕ_2. To estimate the utility parameters, I minimize a quadratic form in the deviations of the sample moments of interest from the corresponding numerically obtained model moments. The model moments depend both on the structural parameters and on the VAR parameters. Joint estimation of (ϕ_1, ϕ_2) with Hansen's (1982) GMM requires solving the model by discretizing the state space for each evaluation of the objective function and is computationally too burdensome. However, a consistent estimate $\hat{\phi}_{2,T}$ of ϕ_2 can be obtained by Ordinary Least Squares, as was done in the previous section. The sampling error in that estimation must be taken into account when ϕ_1 is estimated holding the VAR parameters fixed at $\hat{\phi}_{2,T}$. To see how this is done, let g_{1T} be the difference between the sample moments from the data and the model moments, let g_{2T} be the sample means of the orthogonality conditions corresponding to the VAR, and let $g_T = [g'_{1T}, g'_{2T}]'$. Then the estimator for ϕ_1 satisfies

$$\hat{\phi}_{1,T} = \operatorname{argmin} g_{1T}(\phi_1, \hat{\phi}_{2,T})' W_{T,11} g_{1T}(\phi_1, \hat{\phi}_{2,T}), \tag{9}$$

where the minimization is over $\phi_1 \in P$, a compact set, and where $W_{T,11}$ is a positive definite weighting matrix. Burnside (1991) shows that the optimal choice for $W_{T,11}$ depends on S, the asymptotic variance–covariance matrix of the orthogonality conditions g_T evaluated at the true parameter values and on the derivatives of g_T with respect to both ϕ_1 and ϕ_2. A consistent estimate of S can be constructed as in Newey and West (1987) and consistent estimates of the derivatives of the orthogonality conditions are found by taking numerical and/or analytical derivatives of the sample orthogonality conditions at the parameter estimates. These are the channels through which the sampling error in $\hat{\phi}_{2,T}$ influences the estimation of ϕ_1. Using this optimal weighting matrix, the standard Hansen (1982) test of the over-identifying restrictions remains valid. A more formal and detailed description of this sequential GMM technique can be found in Burnside (1990, 1991).

The parameters were estimated by iterating on the weighting matrix until convergence. Convergence is defined as $\max_{ij}|W_n(i,j) - W_o(i,j)| < 10^{-6}$ with $W_n(i,j)$, $W_o(i,j)$ the elements of the new, respectively old, weighting matrix. Four sets of different starting values led to the same parameter estimates.

3.2. Estimation results

Table 3 contains results for two estimation exercises on the addilog preference specification. The model moments used in the first experiment are the means, variances and first autocovariances of exchange rate changes and of the forward premium together with the covariance between the forward premium at time t and exchange rate changes at time $t+1$. This provides seven orthogonality conditions to estimate the two preference parameters (α, γ). The second experiment uses the means, variances and first autocovariances of exchange rate changes and of the forward bias. The sample moments are computed from quarterly \$/yen rates. The curvature parameters

Table 3

Estimation results for addilog utility.
Sample period: 1975:2–1989:4.

	α	γ	J_T	$\chi^2(1)$
AL I	4.398	1.786	8.928	6.287
	(1.329)	(0.579)	(0.112)	(0.012)
AL II	7.985	3.895	8.112	7.303
	(2.051)	(0.851)	(0.088)	(0.007)

Notes: The first set of moments used in estimation are the means, variances and first autocovariances of currency depreciation and the forward premium and the covariance between currency depreciation and the lagged forward premium (AL I row). The second set of moments include mean, variance and first autocovariance from both the forward bias and exchange rate changes (AL II row). Parameter estimates of the utility parameters of the addilog utility specification (α, γ) are obtained according to the estimation procedure described in text. In computing an estimate for S, the variance–covariance matrix of the orthogonality condtions at the optimum, four Newey–West (1987) lags were used. The J_T statistic has five degrees of freedom for the first set of moments and four for the last set of moments, with asymptotic p-values given in parentheses. The $\chi^2(1)$ test statistic tests the equality of α and γ.

of the utility function are quite precisely estimated, and are in the 'admissible range' proposed by Mehra and Prescott (1985).[8] A Wald test rejects the hypothesis that the estimated values for α and γ are equal at the 5 percent level. The over-identifying restrictions are not rejected at the 5 percent level for either estimation exercise, but they would be at the 10 percent level for the second set of moments. Given the small sample size, the failure to reject the over-identifying restrictions may reflect the low power of the test, rather than substantial evidence in favor of the models.

The implied moments at the parameter estimates are compared with the sample moments in table 4 (columns 1 through 3). Two features of the data stand out. First, exchange rate changes and the forward bias are far more variable and less persistent than the forward premium. To facilitate interpretation, note that the forward premium can be written as the difference of the two predictable components in exchange rate changes and the forward bias, i.e. $FP_t = \mathrm{E}_t[DS_{t+1}] - \mathrm{E}_t[FB_{t+1}]$. Hence, this feature of the data is indicative of high variability of the forecast error associated with exchange rate changes and highly autocorrelated predictable components in exchange rate changes and/or the forward bias [see also Macklem (1991)]. The model's first-order correlation coefficient for all three series is always within two standard errors of the data moment, but the model tends to under-predict exchange rate and

[8] A value of 10 is the upper limit of their 'admissible range' for the coefficient of relative risk aversion.

Table 4. Implied moments and sensitivity analysis. Sample period: 1975:2–1989:4.

		Addilog utility					Homothetic utility		
	Sample mom.	AL I	AL II	SENS I	SENS II	EXTR	SENS I	SENS II	EXTR
$E[DS]$	5.821	0.649*	−0.761*	−18.726	−0.474*	−0.764*	−2.252	−2.227	−2.252
	(3.989)			19.695			−2.252		
$\sigma[DS]$	26.033	7.951	14.468	4.758	20.481	14.509	4.516	6.387	4.516
	(2.639)			22.759*			4.516		
$AC[DS]$	0.164	0.266*	0.146*	0.105**	0.146**	0.145**	0.486	0.486	0.486
	(0.122)			0.466			0.486		
$E[FB]$	2.361	0.082**	0.271**	0.013**	0.542**	0.706**	−0.008**	0.021**	0.010**
	(4.402)			0.796**			0.020**		
$\sigma[FB]$	27.147	6.796	13.235	3.835	18.737	13.279	3.684	5.211	3.684
	(2.305)			21.283*			3.684		
$AC[FB]$	0.221	0.000*	0.000*	0.000*	0.000*	−0.000*	−0.000	−0.000**	0.001**
	(0.118)			0.000*			0.000*		
$E[FP]$	3.481	0.566	−1.031	−18.739	−1.016	−1.470	−2.272	−2.248	−2.263
	(0.748)			18.899			−2.244		
$\sigma[FP]$	3.289	4.127**	5.841	2.809**	8.261	5.825	2.611**	3.694**	2.609**
	(1.112)			8.417			2.612*		
$AC[FP]$	0.721	0.553*	0.567*	0.393	0.567*	0.567*	0.486	0.486	0.486
	(0.100)			0.617*			0.486		
b	−2.217	1.000	1.001	1.000	1.001	1.003	1.000	1.000	1.001
	(0.829)			1.002			1.000		
$\sigma[RP]$	10.622	0.011	0.037	0.002	0.075	0.131	0.002	0.007	0.004
				0.102			0.005		

Notes: The first column reports the sample moments for \$/yen rates. The data are described in the data appendix. The sample frequency is quarterly and the data are annualized, i.e. multiplied with 400. DS refers to exchange rate changes, FB to forward bias, FP to forward premium and RP to risk premium, all as defined in the text. σ refers to the standard deviation, AC to the first-order autocorrelation. The b entry refers to the slope coefficient of a typical unbaisedness regression, i.e. regressing exchange rate changes onto a constant and the forward premium. Its standard error is robust to heteroskedasticity. The standard deviation of the risk premium is based on the fitted value from that regression. The standard errors for the sample moments are obtained by viewing estimation of these statistics as a method of moments estimation problem and applying the correponding asymptotic distribution theory coupled with the delta method. The maximum order of Newey–West (1987) weights is 4.

The second and third columns contain the implied moments at the estimated parameter values (see table 3). The b coefficient here is computed as the covariance of the forward premium at time t and exchange rate changes at time $t+1$ divided by the variance of the forward premium, all of which can be derived from the stochastic structure of the model. SENS I stands for the sensitivity experiment where the model is simulated over a range of parameters, $\alpha \in [1.5, 12.5]$, $\gamma \in [0.5, 6.0]$ in increments of 0.5 for addilog utility and $\delta \in \{0.25, 0.50, 0.75\}$, $\alpha \in [0, 15.5]$ in increments of 0.5 for homothetic utility. β is fixed at 0.99. The minimum (maximum) moment over the range is reported on the first (second) line. SENS II refers to the experiment in which the innovation variances of the exogenous processes are doubled while keeping their correlation structure intact. The parameters for addilog utility are the ones estimated with the second set of moments, the parameters for homothetic utility are $\delta = 0.50$, $\alpha = 8.0$. The EXTR columns refer to an experiment at the same parameters as just mentioned but with an added 'crash state' for the consumption growth processes in which they fall 50 percent below their unconditional means.

The asterisks are to be interpreted as follows: *Within 2 standard errors of sample moment. **Within 1 standard error of sample moment.

forward bias volatility and over-predict forward premium volatility. Nevertheless, the latter is within one standard error of the data moment in the first experiment.

The second striking feature in the data is the firm rejection of the unbiasedness hypothesis. The b coefficient in table 3 is the slope coefficient of a regression of exchange rate changes onto a constant and the forward premium.[9] The regression estimated is

$$DS_{t+1} = a + bFP_t + e_{t+1}. \tag{10}$$

The constant a is estimated to be 13.493 with a heteroskedasticity-consistent standard error of 4.159. The slope coefficient b is estimated to be -2.217 with a standard error of 0.829. Under the null either of a no risk premium or a constant risk premium, the regression coefficient b should equal 1. Not only is the hypothesis rejected, b is also significantly negative. With FB_{t+1} on the left-hand side, the slope coefficient of the regression in (10) would be $b-1$. The standard deviation of the fitted value of such a regression provides a lower bound on the standard deviation of the risk premium in the forward market. It amounts to over 10 percent on an annualized basis. The mean of the risk premium, which equals the mean of the forward bias, is not significantly different from zero. The big challenge for the model therefore lies in matching the variability not the mean of the risk premium.

The standard deviation of the risk premium produced by the model is only 0.011 (0.037) when the first (second) set of moments is used. As a consequence, the implied forward bias, which decomposes into the risk premium and a serially uncorrelated forecast error, is virtually serially uncorrelated and the implied b coefficient virtually equals 1.0 in both estimation experiments. Note that the first experiment includes both the covariance of the forward premium (at time t) and exchange rate changes (at time $t+1$) and the variance of the forward premium, the ratio of which equals b. Nevertheless, the best overall fit still leaves us with a world in which unbiasedness approximately holds.

Finally, at the estimated parameter values the CIA constraints bind and the model predicts unitary velocity. The failure of the model to produce non-binding CIA constraints at reasonable parameter values is the main reason that no estimation is attempted for the homothetic preference specification. With homothetic preferences, binding CIA constraints imply that exchange rates do not depend on preference parameters at all and that the forward premium is also not very sensitive to utility parameter changes (see the expressions in table 1). Hence, the moments of interest are not informative about preference parameters and the objective function surface in the

[9] For a recent evaluation of this particular unbiasedness test, see Bekaert and Hodrick (1993).

estimation would be virtually flat. Some simulation experiments confirming this conjecture are reported below.

3.3. Sensitivity analysis

In table 4 (columns 4 through 9) I report the results of various simulation experiments. First, I examine the robustness of the results discussed above to changes in the utility parameters. The economy is simulated over parameters in a roughly two standard error band around the estimated values, i.e. $\alpha \in [1.5, 12.5]$ and $\gamma \in [0.5, 6.0]$ in increments of 0.5 for addilog utility. For homothetic utility, I choose $\delta \in \{0.25, 0.50, 0.75\}$ and $\alpha \in [0, 15.5]$ in increments of 0.5. The minimum (maximum) moment over the range is reported on the first (second) line in the columns indicated by SENS I. The second experiment (SENS II) doubles the innovation variances of the exogenous processes while keeping their correlation structure intact. In the last experiment, denoted EXTR, the utility parameters are fixed at the same values as in SENS II but a 'crash state' is added to the stochastic structure of the economy, as in Rietz (1988) and Backus et al. (1992). In the crash state, home and foreign consumption growth fall 50 percent below their unconditional means, whereas money growth rates are put equal to their unconditional means. The state is given an extremely low unconditional probability (<0.00001) such that it does not affect the correlation structure of the shocks. The probability that the crash state persists is smaller than 0.001.

The mean of exchange rate changes is very sensitive to changes of the curvature parameters in the addilog preference specification. This is not surprising: when the intertemporal elasticity of substitution for the foreign good is very low (high γ), a substantial appreciation of the home currency is needed to make the representative agents willing to hold the growing foreign good endowment and vice versa. At high curvature parameters, the volatility of exchange rate changes is matched but at these parameter values the standard deviation of the forward premium is unrealistically high. With both utility functions, the model never comes within one standard error of the first-order correlation coefficient of the forward premium. The lack of persistence in the model's forward premium is much more pronounced in Macklem's (1991) simulations of a Lucas-type economy.

The model's failure to generate a significantly variable risk premium is robust to parameter changes. Recall that a lower bound on the standard deviation of the risk premium amounted to about 10 percent. The best the Svensson model can do over the simulated parameter range is a risk premium that is almost 100 times less variable. Consequently, the implied slope coefficients from simple unbiasedness tests are not different from 1 up to two decimal points.

As derived in the previous section, if the CIA constraints are binding, the exchange rate only depends on monetary factors in the homothetic case. The CIAs always bind and, clearly, money supplies are not variable enough to account for exchange rate behavior. The risk premium is now driven by the correlation between monetary shocks and the real shocks present in the expression for n_{t+1}. The mean and variance increase with α, but the effects are small, and variability in the theoretical risk premium remains minuscule.

More variable exogenous processes in the SENS II experiment lead to 50 percent increases in volatility of both exchange rate changes and the forward premium. The mean of the risk premium increases, but even in an economy with endowment shocks twice as variable as aggregate consumption data, the variability of the risk premium remains very low.

The EXTR experiment builds a peso problem into the stochastic structure of the economy. The crash state is an extreme event which agents take into account when forming expectations, but is very unlikely to occur in small samples. This renders the usual procedures of statistical inference invalid. Many researchers have suggested the peso problem as a plausible explanation for the negative relationship between the forward premium and exchange rate changes in regression tests. Although risk premium volatility quadruples in the addilog case, the b coefficient increases. If the persistence of the crash state is increased slightly, the effects are more dramatic, but b remains above 1. The effects of introducing extreme states for the money processes are not as strong and are not reported.[10]

Two results seem to be robust to quite drastic perturbations of preference parameters or the law of motion of the forcing processes. First, exchange rate and forward premium volatility cannot be matched simultaneously. Second, the standard deviation of the risk premium is far too low. Hence, the more complicated risk premium expression in the Svensson model does not help overturn the negative results of Macklem (1991), Backus et al. (1992) and Engel (1992) for the Lucas model.

One potential reason is its failure to deliver variable velocity. From eq. (6), the inverse of home velocity for respectively addilog and homothetic utility is given by

$$\begin{aligned} K(\theta_t) &= \max\left\{1, \beta \mathrm{E}_t\left[K(\theta_{t+1})\,\frac{gx_{t+1}^{1-\alpha}}{\omega_t}\right]\right\}, \\ K(\theta_t) &= \max\left\{1, \beta \mathrm{E}_t\left[K(\theta_{t+1})\,\frac{(gx_{t+1}^{\delta}gy_{t+1}^{1-\delta})^{1-\alpha}}{\omega_t}\right]\right\}. \end{aligned} \tag{11}$$

[10]Backus et al. (1992) find that the combination of habit-forming utility with an extreme state for exchange rate changes in a Lucas-type economy does significantly affect the slope coefficient in an unbiasedness regression.

With addilog utility, only home (foreign) consumption and money growth matters for the CIA on home (foreign) consumption. With homothetic utility there is interaction between home and foreign consumption processes. High β's and low money and consumption growth are seen to be conducive to variable velocity. The economic intuition is clear. An expected monetary contraction makes it more attractive to keep currency in order to buy goods tomorrow at potentially deflated prices. Likewise, with $\alpha(\gamma)$ bigger than 1, the desire for consumption smoothing drives down the marginal utility of cash today if expected consumption growth is low.

Only in the case of a crash state for the consumption processes did a state occur where the CIA constraint was slack. For reasonable parameter values, the international Svensson economy seems to be unable to generate deviations from unitary velocity. This, of course, strengthens the closed-economy results of Hodrick et al. (1991).

4. Further interpretation of the results

4.1. Hansen–Jagannathan bounds

In section 2, the IMRS was shown to be a crucial determinant of the risk premium in the forward market. Hansen and Jagannathan (1991) show that projecting n_{t+1} onto a space of asset payoffs gives rise to a mean–variance frontier that $(\sigma[n], \mathrm{E}[n])$ have to satisfy. When attention is restricted to the space of excess returns, one can show [see, for instance, Bekaert and Hodrick (1992, p. 503)] that

$$\frac{\sigma[n]}{\mathrm{E}[n]} \geqq (\mathrm{E}[R]'\Sigma^{-1}\mathrm{E}[R])^{1/2}, \tag{12}$$

where $\mathrm{E}[R]$ is the vector of expected excess returns on a set of assets and Σ is the variance–covariance matrix of the excess returns. The risk-adjusted mean return on the right-hand side is known in finance as the (generalized) Sharpe ratio. It is the ratio of the mean return to standard deviation of the optimal portfolio formed from the set of assets in a mean–variance framework. The left-hand side of eq. (12) is the coefficient of variation of the nominal IMRS.

To generate the sample analogues of the Sharpe ratio in eq. (12), think of the forward bias as an ordinary excess return. It is the percentage excess dollar return from the portfolio that buys a unit of the foreign currency in the forward market then sells the unit at the (future) spot rate while simultaneously investing S_t dollars at the dollar interest rate. Its Sharpe ratio, the absolute value of the mean of the excess return divided by its standard deviation, is 0.030 (with a standard error of 0.160). The predictability of the forward bias by the forward premium could in principle be

Table 5

Coefficients of variation of IMRS.
Sample period: 1975:2–1989:4.

	$\alpha=2$	$\alpha=4$	$\alpha=8$	$\alpha=10$	$\alpha=25$	$\alpha=50$	$\alpha=100$
AL	0.010	0.016	0.030	0.038	0.096	0.193	0.388
HO	0.009	0.015	0.031	0.039	0.100	0.203	0.420

Notes: For addilog utility (AL) γ is put equal to α, for homothetic utility (HO) I let $\delta=0.50$. The discount factor is fixed at 0.99. The coefficient of variation of the IMRS is computed with the same solution method as all other model moments.

exploited to improve the risk–return trade-off, for instance by using a trading rule. Hansen and Jagannathan (1991) demonstrate that one can incorporate conditioning information by creating a pseudo-return, which is the excess return scaled with a variable that predicts it. In this case, the pseudo return is the forward bias scaled by the lagged forward premium. The Sharpe ratio for a portfolio consisting of the yen forward bias and the pseudo return is 0.566 with a standard error of 0.115.[11] The improvement in the risk–return trade-off when the scaled return is included is dramatic.

In table 5 I report the coefficients of variation of the dollar IMRS for various parameter combinations. Comparison of the coefficients of variation with the Sharpe ratio reported above reveals that the Svensson model does not pass the Hansen–Jagannathan test. Only when α exceeds 50 does the coefficient of variation move into a two standard error region around the bound implied by the portfolio incorporating conditioning information in the forward premium.

4.2. *The impact of habit persistence*

The Hansen–Jagannathan test suggests the importance of the variability of the IMRS to solving the forward market puzzle. Backus et al. (1992) show that introducing habit-forming utility in a Lucas-type model substantially increases the variability of the IMRS and the risk premium. As Hodrick et al. (1991) find that habit-forming utility induces quite variable velocity, it seems useful to investigate whether this result carries over to the Svensson economy.

Let the service flow of consumption at time t be the difference between consumption purchases today and a fraction h of consumption purchases of the previous period. I constrain h to be the same for the home and foreign

[11]The standard errors for the Sharpe ratio are computed in the same way as the standard errors for the sample moments in table 4. Similar high Sharpe ratios are found by Bekaert and Hodrick (1992), who use international stock market and forward market data, and by Backus et al. (1992), who use an investment rule based on the unbiasedness regression.

Table 6

The impact of habit persistence.
Sample period: 1975:2–1989:4.

	$h=0.0$	$h=0.25$	$h=0.50$	$h=0.75$	$h=0.80$	$h=0.85$
Panel A: Addilog utility						
$\sigma[DS]$	14.468	14.940	18.011	24.851	90.331	212.659
$\sigma[RP]$	0.037	0.071	0.071	0.205	0.440	3.544
$cv[IMRS]$	0.030	0.032	0.038	0.056	0.063	0.133
Panel B: Homothetic utility						
$\sigma[DS]$	6.026	5.864	5.897	6.942	40.409	48.789
$\sigma[RP]$	0.005	0.005	0.008	0.028	0.069	0.366
$cv[IMRS]$	0.032	0.032	0.033	0.047	0.074	0.238

Notes: For addilog utility the parameters are $\alpha=7.985$, $\gamma=3.895$ as estimated in table 3, for homothetic utility $\delta=0.50$, $\alpha=8.0$. The service flow derived from home and foreign goods is consumption today minus h times consumption yesterday. cv stands for coefficient of variation, σ for standard deviation.

goods. Some simulation results are reported in table 6. As h is increased, the representative agent becomes implicitly more risk averse and the variabilities of the IMRS, exchange rate changes and the risk premium go up. At high h's, CIA constraints do not bind in more than half of the states and the Svensson model generates variable velocity. Unfortunately, the effect on the risk premium becomes only of the order of magnitude needed when h is increased above 0.8. Large values of h are consistent with the estimates of habits in the closed-economy models of Ferson and Constantinides (1991) and Heaton (1991). However, the model then produces extremely variable and negatively autocorrelated forward premiums and exchange rates.

Given the results here and in the previous section, the forward market puzzle seems to constitute an even greater challenge for consumption-based asset pricing models than does the equity risk premium puzzle in closed economies. Constantinides (1990), using habit persistence, and Kandel and Stambaugh (1988) and Kocherlakota (1990), using somewhat extreme parameter values, have shown that a simple real consumption-based asset pricing model is consistent with the high mean excess return in the stock market. Of course, the equity premium puzzle is an unconditional moment puzzle, whereas the forward market puzzle involves conditional moments since it is a consequence of the predictability of exchange rate changes.

4.3. *The potential impact of PPP deviations*

One problem with the above modelling strategy is that it incorporates an implicit assumption of PPP. To see this, note that the law of one price holds for both tradable goods and that tastes are the same across countries, consequently PPP holds in this model. PPP deviations could potentially be an important factor contributing to risk premium variability. For ease of exposition, I utilize the logarithm of the relevant variables. Let $FB^*_{t+1} = (S_{t+1} - F_t)/F_t$ so that $fb_{t+1} = \ln(S_{t+1}) - \ln(F_t) \approx FB^*_{t+1}$ and let $rp_t = \mathrm{E}_t[fb_{t+1}]$.[12] By covered interest parity in continuously compounded form, the log of the forward premium equals the interest differential between the United States and Japan. By adding and subtracting inflation in both countries, one can derive:

$$fb_{t+1} \equiv \Delta q_{t+1} + r^*_{t+1} - r_{t+1}, \tag{13}$$

where Δq_{t+1} are (logarithmic) real exchange rate changes, and r^*_{t+1} (r_{t+1}) the ex post real interest rate in Japan (the United States), defined as the nominal interest rate minus the inflation rate. Taking expectations, eq. (13) decomposes the risk premium into expected real exchange rate changes and expected real interest differentials [see also Korajczyk (1985)]. In models based on PPP, real exchange rate changes are set to zero and the risk premium is totally driven by real interet rate differentials.

If the real exchange rate is a martingale, i.e. $\mathrm{E}_t[\Delta q_{t+1}] = 0$, the PPP assumption would in fact be a harmless simplifying assumption. Meese and Rogoff (1988) provide some evidence in favor of this hypothesis, while Korajczyck (1985), making the martingale assumption, empirically links risk premiums on various currencies to real interest differentials with some success. Recent evidence in Huizinga (1987) and Cumby and Huizinga (1991), however, suggests that real exchange rate changes contain a substantial predictable component. If this is true, the variability of the risk premium is related both to the variability of ex ante PPP deviations and of ex ante real interest differentials.

Without attempting to settle this controversial issue here, table 7 offers an informal estimate of the relative contributions of expected real exchange rate changes and real interest differentials. To obtain estimates of expected values, I simply projected fb_{t+1}, Δq_{t+1}, and $r^*_{t+1} - r_{t+1}$ onto a number of information variables consisting of the lagged real exchange rate, the lagged forward premium and the lagged inflation differential. The variability of the risk premium can then be decomposed into the variability of its two components and their covariance using the variance of the fitted values in the regressions. The table reveals that the explained part, i.e. the predictability of real interest differentials, is indeed higher than the predictability of real exchange rates. However, the total variance of real interest differentials is

[12] It is well known that inference in unbiasedness tests and hence the implied risk premiums are not sensitive to the use of logs vs. levels [see, for example, Hansen and Hodrick (1983)].

Table 7

Relative variability of expected PPP deviations and real interest differentials.
Sample period 1975:2–1989:4.

	fb	Δq	r^*-r	$(\Delta q, r^*-r)$
$\sigma^2[\cdot]$	696.093	647.063	10.380	19.325
$\sigma^2[E_t[\cdot]]$	122.703	81.059	4.853	18.396

Notes: The symbols fb, Δq, and r^*-r indicate, respectively, the (logarithmic) forward bias, real exchange rate changes and the real interest differential between the U.S. and Japan as described in the text. The deflators used to compute inflation rates in the U.S. and Japan are the deflators from the consumption series used as exogenous processes and they were deseasonalized by regressing on four seasonal dummies. σ^2 denotes the sample variance. The conditional expectation $E_t[\cdot]$ is computed from the projection of the variable in the column onto a constant, lagged real exchange rate changes, the lagged forward premium and the lagged inflation differential. The last column contains the sample covariance between (expected) real exchange rate changes and the (expected) real interest differential.

dwarfed by the total variance of real exchange rate changes. According to the table, about three-quarters of the variability of the forward market's risk premium is accounted for by expected real exchange rate changes.

This result indicates that most of the literature might be missing an important factor in the determination of the risk premium. However, at reasonable parameter values the model here (and others in the literature) are not able to deliver even the part of the variability of the risk premium that is due to real interest differentials. Hence, the puzzle might not be solved by successfully introducing PPP deviations alone.

5. Conclusions

This paper demonstrates that, given the short data sample at hand, Svensson's CIA model cannot be statistically rejected with respect to its overall fit for some selected moments of exchange rate changes, the forward bias and forward premium. Nevertheless, it is clear that it is unable to mimic some salient features of foreign exchange market data. Specifically, exchange rate variability can be matched with time separable preferences, but the utility parameters needed for this imply a variability of the expected return in the forward market that is several orders of magnitude lower than what can be inferred from the data. The empirical predictions of the Svensson model therefore look very similar to those of the Lucas model, despite the forward-looking character of exchange rates and the potential for variability of velocity in the Svensson model.

It seems that a more drastic perturbation to the present framework is needed to account for the deviations from the unbiasedness observed in the data. In Bekaert (1992), agents take decisions at the weekly frequency, forcing processes are conditionally heteroskedastic and the preferences combine short-run durability with long-run habit persistence as in Heaton (1991). The model matches the relative persistence and variability of exchange rate changes and the forward premium and also substantially increases the variability of the risk premium. Yet, it falls short of the variability observed in the data. The model still imposes PPP, however, and section 4 of this paper revealed the need to incorporate PPP deviations in models of the risk premium.

A fruitful avenue for further research, might be to combine 'policy regime shifts' and 'learning' [see Kaminsky (1988), Lewis (1989), Engel and Hamilton (1990)]. Rational agents' decisions are influenced by the policy regime which they might not know with certainty at all times. It is conceivable, for instance, that agents only gradually learn about a regime shift and in the mean time are likely to make faulty exchange rate predictions.

Data appendix

The exchange rate data set used in this paper is also used in Bekaert and Hodrick (1992, 1993). The original data are daily bid and ask spot and forward rates, obtained from Citicorp Database Services. The data are captured from a Reuter screen and represent quoted market prices. Filter tests were run to check for errors, and the errors were corrected with observations from the International Monetary Market Year Book or the *Wall Street Journal*. All data used are averages of bid and ask rates sampled at the end of the quarter. For the robustness of empirical results on unbiasedness to transaction costs and alignment of the data, see Bekaert and Hodrick (1993).

Money supplies were obtained from the International Financial Statistics (IFS) data tape. I aggregate the series 34 (money) and 35 (quasi-money) to obtain a broad money concept. Due to the introduction of MMDAs and super NOW accounts, there is an outlier in the U.S. data in the first quarter of 1983. I replaced the money growth, which was five standard errors above the mean, by a growth rate two standard errors above the mean.

Consumption data are taken from the OECD Quarterly National Accounts. The consumption series is obtained by adding real expenditures on nondurables and services.

Both the money and real consumption data were divided by total population (series 99z in the IFS data set) to arrive at per capita data. These population data are mid-year estimates which are linearly interpolated to obtain quarterly data. After taking growth rates, all the series are deseasonalized by regressing the demeaned series on four quarterly dummies.

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